On the cosmological significance of oscillons

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Time evolution of spherically symmetric configurations is investigated in Einstein's theory of gravity. The geometry of the spacetime is represented by the line element

$$ds^2 = \Omega\, d\tau^2 - \Omega\, d\rho^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2),$$

where Ω, r are smooth functions of the coordinates τ and ρ.

The matter content is chosen to be a composition consisting of a perfect fluid (representing the cosmological background) and a self-interacting scalar field:

$$T^{\text{fluid}}_{\alpha\beta} = (e+p)\,u_\alpha u_\beta - p\, g_{\alpha\beta}$$

$$u^\alpha = \frac{1}{\sqrt{\Omega}}(\cosh y,\ \sinh y,\ 0,\ 0),$$

$$\text{EOS:} \quad p = \gamma e, \quad \gamma = 1/3,$$

$$T^{\psi}_{\alpha\beta} = \nabla_\alpha\psi\nabla_\beta\psi - \frac{1}{2}g_{\alpha\beta}[g^{\mu\nu}\nabla_\mu\psi\nabla_\nu\psi - V(\psi)],$$

Self-interaction is given by the potential

$$V(\psi) = c\,(\psi^2 - \psi_0^2)^2.$$

Evolution is determined by the following first order symmetric hyperbolic system:

$$\partial_\tau r = r_\tau$$

$$\partial_\tau r_\tau = \partial_\rho r_\rho + 4\pi r\,\Omega(T^0_0 + T^1_1) - \frac{2m\Omega}{r^2}$$

$$\partial_\tau r_\rho = \partial_\rho r_\tau$$

$$\partial_\tau \Omega = \Omega_\tau$$

$$\partial_\tau \Omega_\tau = \partial_\rho\Omega_\rho + 8\pi r\,\Omega^2(T^2_2 + T^3_3 - T^0_0 - T^1_1) + \frac{\Omega_\tau^2 - \Omega_\rho^2}{\Omega} + \frac{4m\,\Omega^2}{r^3}$$

$$\partial_\tau \Omega_\rho = \partial_\rho \Omega_\tau$$

$$\partial_\tau m = 4\pi r^2(r_\tau T^1_1 - r_\rho T^1_0)$$

$$\partial_\tau \psi = \psi_\tau$$

$$\partial_\tau \psi_\tau = \partial_\rho\psi_\rho - 2\frac{\psi_\tau r_\tau - \psi_\rho r_\rho}{r} - \frac{1}{2}\Omega\frac{\partial V}{\partial\psi}$$

$$\partial_\tau \psi_\rho = \partial_\rho \psi_\tau$$

$$e_\tau - p_\tau \tanh^2 y = -(e_\rho - p_\rho)\tanh y - \frac{e+p}{\cosh^2 y}\left(y_\rho + \frac{\Omega_\tau}{2\Omega}\right)$$

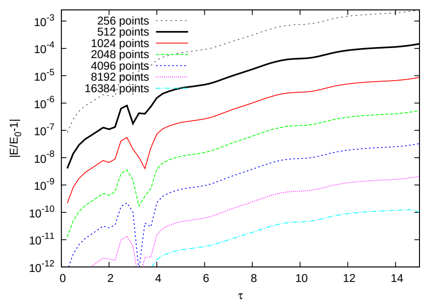
$$2(e+p)\frac{r_\rho \tanh y + r_\tau}{r}$$

$$y_\tau = y_\rho \tanh y - \frac{\Omega_\tau \tanh y + \Omega_\rho}{2\Omega} - \frac{p_\tau \tanh y + p_\rho}{e+p},$$

with four constraint equations solved in the initial state.

Numerical accuracy

The above system was solved numerically using a fourth order accurate finite difference code (*GridRipper* AMR). Besides cosmological scenarios, the system was also tested in simulations of an oscillon field near gravitational collapse (in non-expanding spacetime, without fluid: *e*=0). The following plot shows energy conservation – determined by the use of the Kodama vector field – in the latter case, demonstrating the efficiency of the numerical method:



Why did inflation stop?

Initial data:

$$r(0,\rho) = R_0 \sin\rho, \quad r_\tau(0,\rho) = \sqrt{r_\rho^2 - \Omega\left(1 - \frac{2\mathrm{m}}{r}\right)}$$

$$\Omega(0,\rho) = R_0^2, \quad \Omega_\tau(0,\rho) = \partial_\tau\left(\left(1 - \frac{2\mathrm{m}}{r}\right)^{-1}(r_\rho^2 - r_\tau^2)\right)_{\tau=0}$$

$$\psi(0,\rho) = \psi_2 + (\psi_1 - \psi_2)\left[\exp\left(\frac{-\rho^2}{\rho_0^2}\right) + \exp\left(\frac{-(\rho-\pi)^2}{\rho_0^2}\right)\right]$$

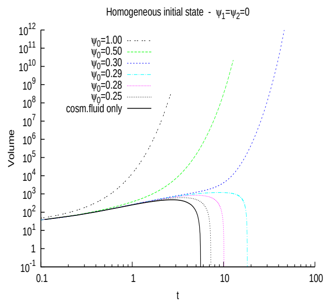
$$\psi_\tau(0,\rho) = 0, \quad e(0,\rho) = e_0, \quad y(0,\rho) = 1.$$

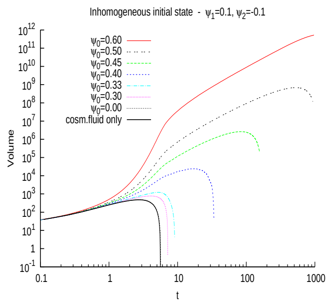
The functional form of *r* indicates that considerations were restricted to the case of closed cosmological models. To have a suitable characterization of the change of the rate of scales the volume

$$Volume = 4\pi\int_o^\pi \sqrt{\Omega}\, r^2\, d\rho$$

of the time level surfaces were plotted against the "averaged" proper time

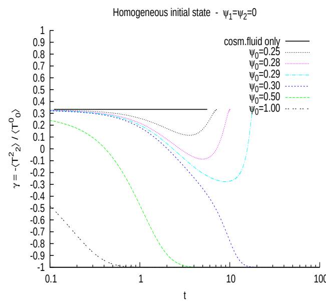
$$t(\tau) = \int_o^\tau\left(\frac{1}{\pi}\int_o^\pi \sqrt{\Omega}\, d\rho\right) d\tau$$

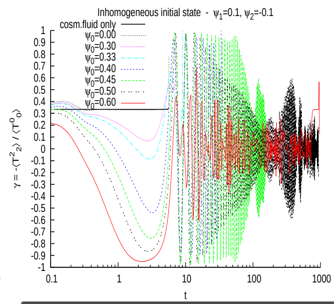




The next figure shows the time dependence of the ratio of the average (tangential) pressure and the energy density.

γ = 1/3	-	radiation dominated universe
γ = 0	-	dustlike
γ < -1/3	-	"superluminal" expansion





A homogeneous universe inflates forever.

Inflation stops automatically as a result of the slight initial inhomogeneity.