

Csizmadia Péter részvételle a gravitációelméleti kutatásokban



“Átmenet”

- 2005 nehézion fizikai kutatások → ált.rel. ?
- Két évvel korábban Fodor Gyulával közösen indítottuk be a numerikus relativitáselméleti programot.
 - Analítikus meggondolások ↔ programozási rutin
 - AMR?



AMR

Csizmadia Péter: TESTING A NEW MESH REFINEMENT CODE IN THE EVOLUTION OF A SPHERICALLY SYMMETRIC KLEIN–GORDON FIELD, 2006 Int. J. Mod. Phys. D 15 107–19

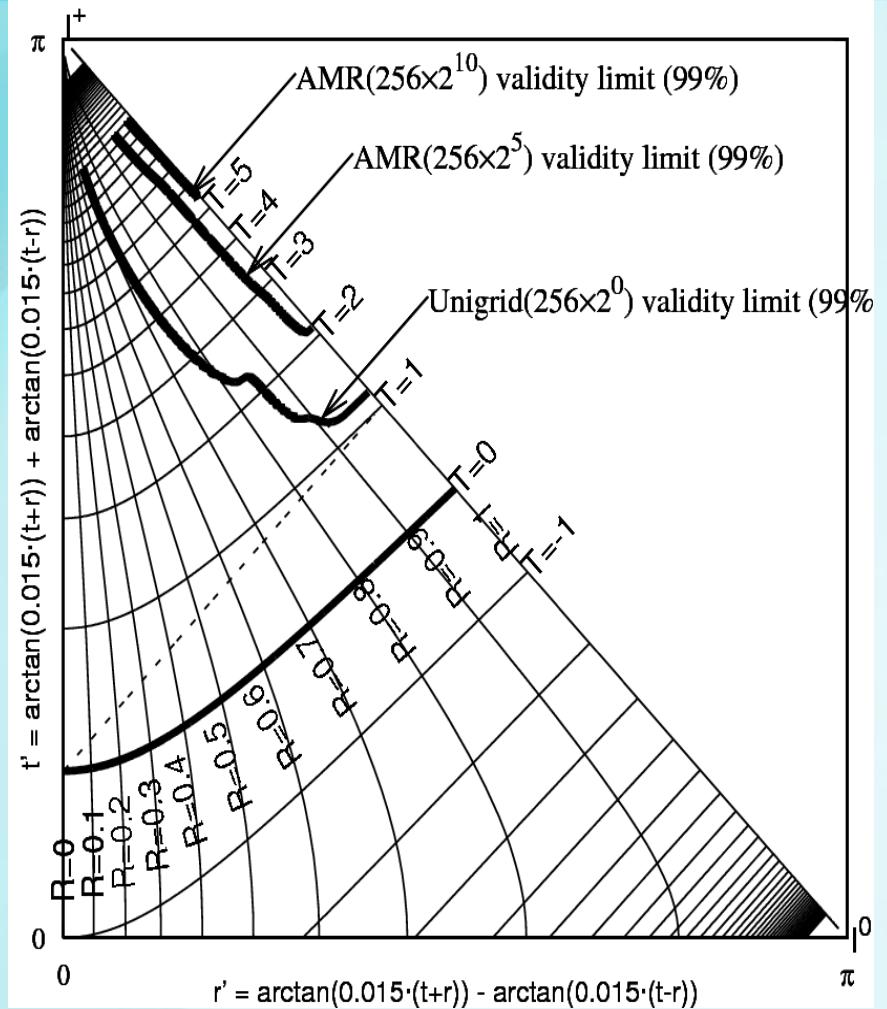
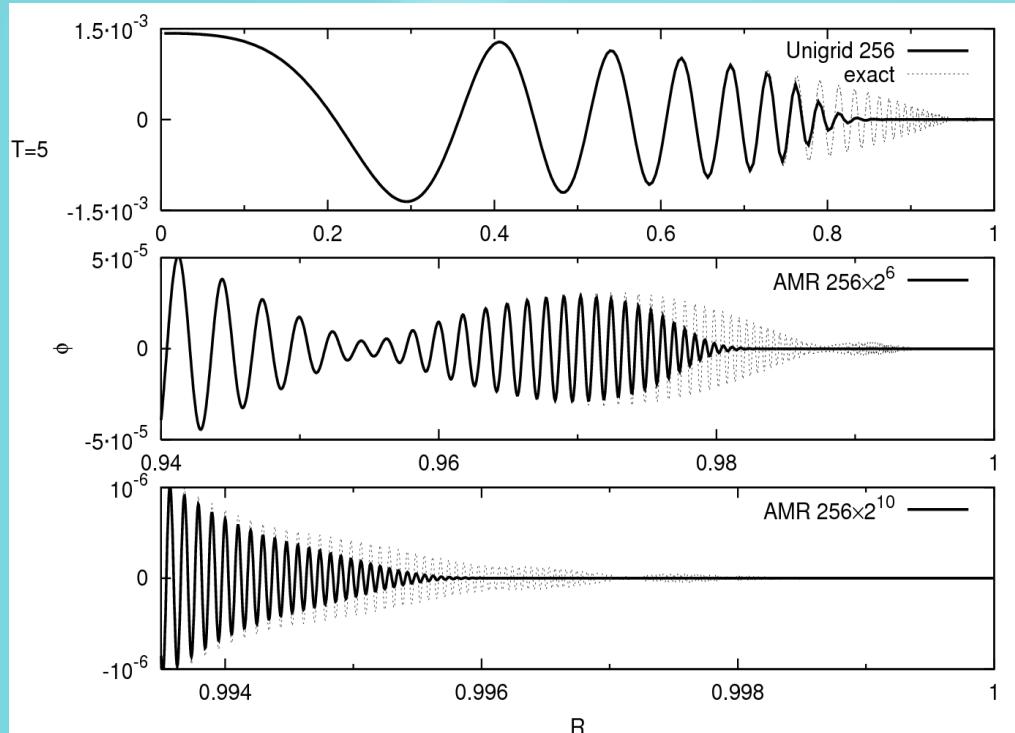
Numerical evolution of the spherically symmetric, massive Klein–Gordon field is presented using a new adaptive mesh refinement (AMR) code with fourth order discretization in space and time, along with compactification in space. The system is non-interacting, thus the initial disturbance is entirely radiated away. The main aim is to simulate its propagation until it vanishes near . By numerical investigations of the violation of the energy balance relations, the space–time boundaries of "well-behaving" regions are determined for different values of the AMR parameters. An important result is that mesh refinement maintains precision in the central region for a longer time even if the mesh is only refined outside of this region. The speed of the algorithm was also tested; in the case of ten refinement levels the algorithm was two orders of magnitude faster than the extrapolated time of the corresponding unigrid run.

Csizmadia Péter: Fourth order AMR and nonlinear dynamical systems in compactified space, Classical and Quantum Gravity 24 (2007) S369-S379

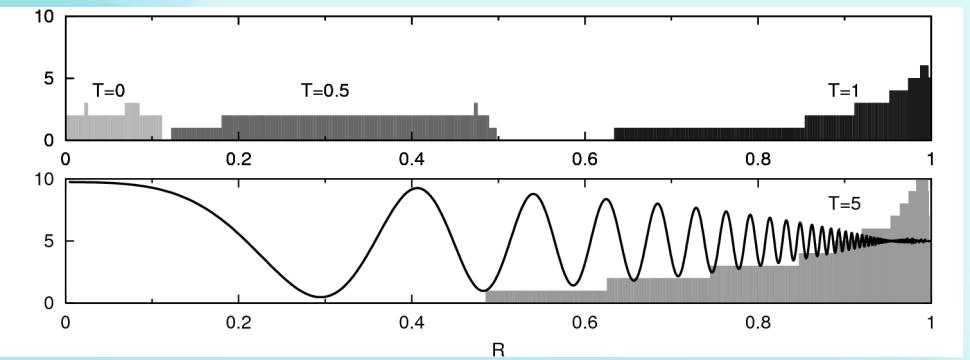
Numerical evolution of the spherically symmetric, massive Klein–Gordon field is presented using a new adaptive mesh refinement (AMR) code with fourth order discretization in space and time, along with compactification in space. The system is non-interacting, thus the initial disturbance is entirely radiated away. The main aim is to simulate its propagation until it vanishes near . By numerical investigations of the violation of the energy balance relations, the space–time boundaries of "well-behaving" regions are determined for different values of the AMR parameters. An important result is that mesh refinement maintains precision in the central region for a longer time even if the mesh is only refined outside of this region. The speed of the algorithm was also tested; in the case of ten refinement levels the algorithm was two orders of magnitude faster than the extrapolated time of the corresponding unigrid run. Time evolutions of certain spherically symmetric systems are investigated where simple explicit second order finite difference methods are not applicable. Due to a compactified space coordinate, efficiency and long-term numerical stability require at least fourth order accuracy for both the massive Klein–Gordon field and the SU(2) Yang–Mills–Higgs system. Moreover, adaptive mesh refinement (AMR) has a crucial role in dealing with high frequency oscillations that appear as an initial disturbance is radiated away. The incompatibility of AMR with fully fourth order accuracy is discussed and a solution is presented. Finally, compactification is compared to standard spherical coordinates and truncated grids in terms of efficiency.

AMR

- Tömeges mezők



- Finomítási szintek



Gridripper & gridsurf

- 2006-2007 Gridripper
- 2008 alkalmazások és az első sikerek

Poster Erice 2008



Gravitációs összeomlások numerikus vizsgálata

Csizmadia Péter

Rácz István

2009. szeptember 3
(KFKI RMKI)

P. Csizmadia and I. Rácz: Gravitational collapse and topology change
in spherically symmetric dynamical systems,
to appear in CQG, **arXiv:0911.2373**

Numerikus Relativitás

Parciális differenciálegyenletek fő megoldási módszerei:

- Véges differenciák (finite difference)
 - diszkretizáció rácson
 - térrács és időlépés szükség esetén finomítható → AMR
 - térben és időben általában másod- vagy negyedrendű pontosság
 - AMR elsőrendő hiperbolikus parc. diff. egyenletek megoldására való
- (Pszeudo)spektrális
 - megoldás közelítése folytonos függvények lineáris kombinációjaként
 - hiperbolikus és elliptikus egyenletekhez is, “végtelen rendű” pontosság
 - nehéz megvalósítani, problémák a diszkontinuitásokkal
- Kevert véges diff. + spektrális (*lásd László András előadását*)

Numerikus Relativitás

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi T_{\alpha\beta}$$

Spherically symmetric GBSSN and its drawbacks

(Generalized Baumgarte-Shapiro-Shibata-Nakamura)

- Works with $\gamma \neq 1 \rightarrow$ one more field than in BSSN
- This field has a characteristic speed that depends on the shift vector.
- 3 metric related and 3 auxiliary variables, 6 evolution equations and 3 constraints.
- Code easily **crashes** due to shock development.
- Impossible to follow evolution inside trapped surfaces.

$$\frac{\partial \chi}{\partial t} = -\frac{2\chi\beta^r}{3} + \beta^r\chi' + \frac{2\alpha\chi K}{3} - \frac{\beta^r\chi\tilde{g}'_{rr}}{3\tilde{g}_{rr}} - \frac{2\beta^r\chi\tilde{g}'_{\theta\theta}}{3\tilde{g}_{\theta\theta}}, \quad (9a)$$

$$\frac{\partial \tilde{g}_{rr}}{\partial t} = -2\alpha\tilde{A}_{rr} + \frac{2\beta^r\tilde{g}'_{rr}}{3} + \frac{4\beta^r\tilde{g}_{rr}}{3} - \frac{2\beta^r\tilde{g}'_{\theta\theta}\tilde{g}_{rr}}{3\tilde{g}_{\theta\theta}}, \quad (9b)$$

$$\frac{\partial \tilde{g}_{\theta\theta}}{\partial t} = \frac{\beta^r\tilde{g}'_{\theta\theta}}{3} - \frac{2\beta^r\tilde{g}_{\theta\theta}}{3} + \frac{\alpha\tilde{A}_{rr}\tilde{g}_{\theta\theta}}{\tilde{g}_{rr}} - \frac{\beta^r\tilde{g}'_{rr}\tilde{g}_{\theta\theta}}{3\tilde{g}_{rr}}, \quad (9c)$$

$$\begin{aligned} \frac{\partial \tilde{A}_{rr}}{\partial t} = & -\frac{2\alpha\tilde{A}_{rr}^2}{\tilde{g}_{rr}} + \frac{4\beta^r\tilde{A}_{rr}}{3} + \alpha K \tilde{A}_{rr} - \frac{\beta^r\tilde{g}'_{rr}\tilde{A}_{rr}}{3\tilde{g}_{rr}} - \frac{2\beta^r\tilde{g}'_{\theta\theta}\tilde{A}_{rr}}{3\tilde{g}_{\theta\theta}} - \frac{\alpha\chi'^2}{6\chi} + \beta^r\tilde{A}'_{rr} - \frac{2\alpha'\chi'}{3} \\ & - \frac{2\chi\alpha''}{3} + \frac{\alpha\chi''}{3} + \frac{2}{3}\alpha\chi\tilde{\Gamma}'\tilde{g}_{rr} - \frac{\alpha\chi\tilde{g}''_{rr}}{3\tilde{g}_{rr}} + \frac{\chi\alpha'\tilde{g}'_{rr}}{3\tilde{g}_{rr}} - \frac{\alpha\chi'\tilde{g}'_{rr}}{6\tilde{g}_{rr}} + \frac{\alpha\chi\tilde{g}''_{\theta\theta}}{3\tilde{g}_{\theta\theta}} + \frac{\chi\alpha'\tilde{g}'_{\theta\theta}}{3\tilde{g}_{\theta\theta}} \\ & - \frac{\alpha\chi'\tilde{g}'_{\theta\theta}}{6\tilde{g}_{\theta\theta}} - \frac{2\alpha\chi\tilde{g}_{rr}}{3\tilde{g}_{\theta\theta}} - \frac{\alpha\chi\tilde{g}'_{rr}\tilde{g}'_{\theta\theta}}{2\tilde{g}_{rr}\tilde{g}_{\theta\theta}} + \frac{2\alpha\chi\tilde{g}'_{rr}^2}{3\tilde{g}_{rr}^2} - \frac{\alpha\chi\tilde{g}'_{\theta\theta}^2}{3\tilde{g}_{\theta\theta}^2}, \end{aligned} \quad (9d)$$

$$\frac{\partial K}{\partial t} = \frac{3\alpha\tilde{A}_{rr}^2}{2\tilde{g}_{rr}^2} + \frac{\alpha K^2}{3} + \beta^r K' + \frac{\alpha'\chi'}{2\tilde{g}_{rr}} - \frac{\chi\alpha''}{\tilde{g}_{rr}} - \frac{\chi\alpha'\tilde{g}'_{\theta\theta}}{\tilde{g}_{rr}\tilde{g}_{\theta\theta}} + \frac{\chi\alpha'\tilde{g}'_{rr}}{2\tilde{g}_{rr}^2}, \quad (9e)$$

$$\begin{aligned} \frac{\partial \tilde{\Gamma}^r}{\partial t} = & -\frac{\beta^r\tilde{g}'_{\theta\theta}^2}{\tilde{g}_{rr}\tilde{g}_{\theta\theta}^2} + \frac{2\beta^r\tilde{g}'_{\theta\theta}}{3\tilde{g}_{rr}\tilde{g}_{\theta\theta}} + \frac{\alpha\tilde{A}_{rr}\tilde{g}'_{\theta\theta}}{\tilde{g}_{rr}^2\tilde{g}_{\theta\theta}} + \beta^r\tilde{\Gamma}'^r + \frac{4\beta^{r''}}{3\tilde{g}_{rr}} - \frac{4\alpha K'}{3\tilde{g}_{rr}} + \frac{\beta^r\tilde{g}''_{\theta\theta}}{3\tilde{g}_{rr}\tilde{g}_{\theta\theta}} - \frac{2\tilde{A}_{rr}\alpha'}{\tilde{g}_{rr}^2} \\ & - \frac{3\alpha\tilde{A}_{rr}\chi'}{\chi\tilde{g}_{rr}^2} + \frac{\beta^r\tilde{g}''_{rr}}{6\tilde{g}_{rr}^2} + \frac{\alpha\tilde{A}_{rr}\tilde{g}'_{rr}}{\tilde{g}_{rr}^3}, \end{aligned} \quad (9f)$$

Primes denote ∂_r . Note that I am following the common practice of using $\tilde{\Gamma}^r$ on the right-hand side only if it appears differentiated.

The Hamiltonian constraint is defined by $\mathcal{H} \equiv K^2 - K_{ab}K^{ab} + R$ and the momentum constraint is defined by $\mathcal{M}_a \equiv D_b K_a^b - D_a K$. With BSSN we also have constraints that arise from the definition of the conformal connection functions: $\mathcal{G}^a \equiv \tilde{\Gamma}^a - \tilde{g}^{bc}\tilde{\Gamma}^a_{bc}$. In spherical symmetry these constraints become

$$\mathcal{H} = -\frac{3\tilde{A}_{rr}^2}{2\tilde{g}_{rr}^2} + \frac{2K^2}{3} - \frac{5\chi'^2}{2\chi\tilde{g}_{rr}} + \frac{2\chi''}{\tilde{g}_{rr}} + \frac{2\chi}{\tilde{g}_{\theta\theta}} - \frac{2\chi\tilde{g}''_{\theta\theta}}{\tilde{g}_{rr}\tilde{g}_{\theta\theta}} + \frac{2\chi'\tilde{g}'_{\theta\theta}}{\tilde{g}_{rr}\tilde{g}_{\theta\theta}} + \frac{\chi\tilde{g}'_{rr}\tilde{g}'_{\theta\theta}}{\tilde{g}_{rr}^2\tilde{g}_{\theta\theta}} - \frac{\chi'\tilde{g}'_{rr}}{\tilde{g}_{rr}^2} + \frac{\chi\tilde{g}'_{\theta\theta}^2}{2\tilde{g}_{rr}\tilde{g}_{\theta\theta}^2}, \quad (10a)$$

$$\mathcal{M}_r = \frac{\chi\tilde{A}'_{rr}}{\tilde{g}_{rr}} - \frac{2\chi K'}{3} - \frac{3\tilde{A}_{rr}\chi'}{2\tilde{g}_{rr}} + \frac{3\tilde{A}_{rr}\chi\tilde{g}'_{\theta\theta}}{2\tilde{g}_{rr}\tilde{g}_{\theta\theta}} - \frac{\tilde{A}_{rr}\chi\tilde{g}'_{rr}}{\tilde{g}_{rr}^2}, \quad (10b)$$

$$\mathcal{G}^r = -\frac{\tilde{g}'_{rr}}{2\tilde{g}_{rr}^2} + \tilde{\Gamma}^r + \frac{\tilde{g}'_{\theta\theta}}{\tilde{g}_{rr}\tilde{g}_{\theta\theta}}. \quad (10c)$$

A straightforward alternative formulation

Coordinates and line element

$$ds^2 = \alpha \beta^2 d\tau^2 - \alpha d\rho^2 - r^2(d\vartheta^2 + \sin^2 \vartheta d\varphi^2)$$

Scalar field

$$T_{\alpha\beta} = \psi_\alpha \psi_\beta - g_{\alpha\beta} \left(\frac{1}{2} g^{\mu\nu} \psi_\mu \psi_\nu - V \right),$$

$$\nabla_\mu \nabla^\mu \psi = -\frac{\partial V}{\partial \psi}.$$

First order strongly hyperbolic system of equations

$$\partial_\tau r_\tau = \beta^2 \partial_\rho r_\rho + 8\pi r \alpha \beta^2 V - \frac{\alpha \beta^2 + r_\tau^2 - \beta^2 r_\rho^2}{r} + \frac{\beta_\tau r_\tau}{\beta},$$

$$\partial_\tau \alpha_\tau = \beta^2 \partial_\rho \alpha_\rho - 8\pi \alpha (\psi_\tau^2 - \beta^2 \psi_\rho^2) + \frac{\alpha_\tau^2 - \beta^2 \alpha_\rho^2}{\alpha} + 2\alpha \frac{\alpha \beta^2 + r_\tau^2 - \beta^2 r_\rho^2}{r^2} + \frac{\beta_\tau \alpha_\tau + \beta^2 \beta_\rho \alpha_\rho}{\beta} + 2\alpha \beta \partial_\rho \beta_\rho,$$

$$\partial_\tau \psi_\tau = \beta^2 \partial_\rho \psi_\rho - 2 \frac{\psi_\tau r_\tau - \beta^2 \psi_\rho r_\rho}{r} + \frac{\beta_\tau \psi_\tau + \beta^2 \beta_\rho \psi_\rho}{\beta} - \alpha \beta^2 \frac{\partial V}{\partial \psi},$$

$$\partial_\tau \psi = r_\tau, \quad \partial_\tau \psi_\rho = \partial_\rho r_\tau, \quad \partial_\tau r = r_\tau, \quad \partial_\tau r_\rho = \partial_\rho r_\tau, \quad \partial_\tau r = r_\tau, \quad \partial_\tau r_\rho = \partial_\rho r_\tau,$$

Constraints to be satisfied on the initial hypersurface

$$\psi_\rho = \partial_\rho \psi, \quad r_\rho = \partial_\rho r, \quad \alpha_\rho = \partial_\rho \alpha.$$

Numerically unstable, truncation errors grow rapidly near $r=0$.

Stabilization using the Misner-Sharp mass as an auxiliary variable

$$m = \frac{r}{\sqrt{1+g^{ab}\partial_a r \partial_b r}} = \frac{r}{2} \left(\frac{\alpha \beta^2 + r_\tau^2 - \beta^2 r_\rho^2}{\alpha \beta^2} \right)$$

First order strongly hyperbolic system of equations

$$\partial_\tau r_\tau = \beta^2 \partial_\rho r_\rho + 4\pi r \alpha \beta^2 (T^\tau_\tau + T^\rho_\rho) - \frac{2m\alpha\beta^2}{r^2} + \frac{\beta_\tau r_\tau + \beta^2 \beta_\rho r_\rho}{\beta},$$

$$\partial_\tau \alpha_\tau = \beta^2 \partial_\rho \alpha_\rho + 8\pi \alpha^2 \beta^2 (T^{\vartheta}_\vartheta + T^\varphi_\varphi - T^\tau_\tau - T^\rho_\rho) + \frac{\alpha_\tau^2 - \beta^2 \alpha_\rho^2}{\alpha} + \frac{\alpha_\tau \beta_\tau + \beta^2 \alpha_\rho \beta_\rho}{\beta} + 2\alpha \beta \partial_\rho \beta_\rho + \frac{4m\alpha^2 \beta^2}{r^3},$$

$$\partial_\tau m = 4\pi r^2 (T^\rho_\rho r_\tau - T^\tau_\tau r_\rho),$$

$$\partial_\tau \psi_\tau = \beta^2 \partial_\rho \psi_\rho - 2 \frac{\psi_\tau r_\tau - \beta^2 \psi_\rho r_\rho}{r} + \frac{\beta_\tau \psi_\tau + \beta^2 \beta_\rho \psi_\rho}{\beta} - \alpha \beta^2 \frac{\partial V}{\partial \psi},$$

$$\partial_\tau \psi = r_\tau, \quad \partial_\tau \psi_\rho = \partial_\rho r_\tau, \quad \partial_\tau r = r_\tau, \quad \partial_\tau r_\rho = \partial_\rho r_\tau, \quad \partial_\tau r = r_\tau, \quad \partial_\tau r_\rho = \partial_\rho r_\tau,$$

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$$\partial_\rho m = 4\pi r^2 (T^\tau_\tau r_\rho - T^\tau_\rho r_\tau),$$

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Comparison with GBSSN

GBSSN

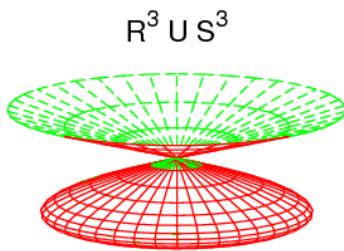
- 3 metric and 3 auxiliary variables.
- **Complicated** system of equations:
 - ◆ 6 nontrivial evolution equations
 - ◆ 3 nontrivial constraints
- **No** time evolution inside trapped surfaces.
- **Crashes** easily due to shock development.

Our formalism

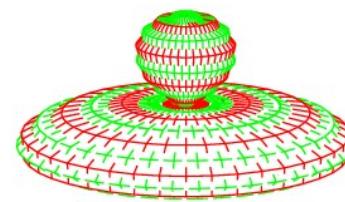
- 2 metric and 1 (2) auxiliary variables + their derivatives
- **Simple** system of equations
 - ◆ 3 nontrivial evolution equations
 - ◆ 1 nontrivial constraint equation
 - ◆ 6+3 trivial equations
- Time evolution **inside trapped surfaces** also, until singularity is reached.
- **Stable.**

Topológiaváltás?

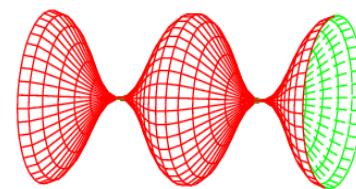
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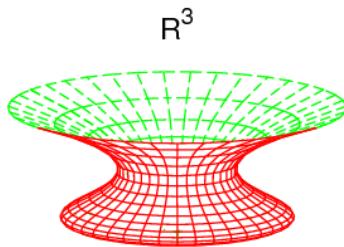
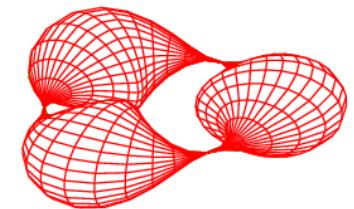
$S^3 \cup S^3$



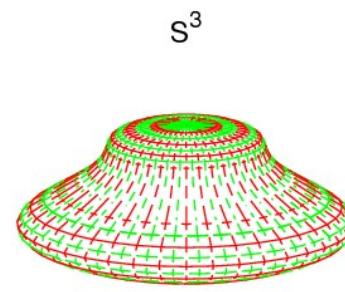
$R^3 \cup S^3 \cup R^3$



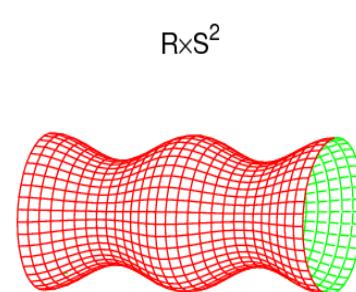
$S^3 \cup S^3 \cup S^3$



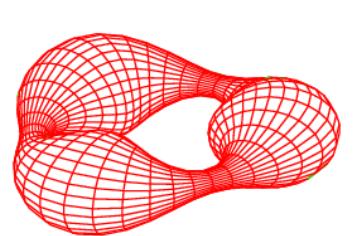
R^3



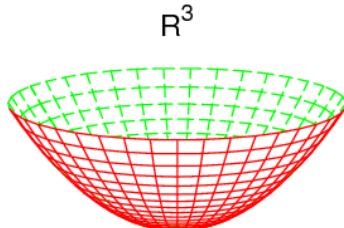
S^3



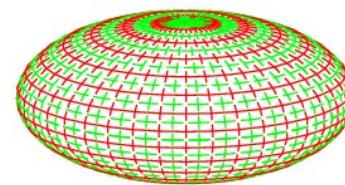
$R \times S^2$



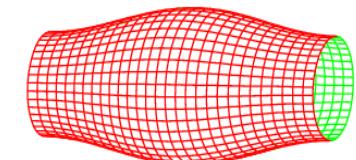
$S^1 \times S^2$



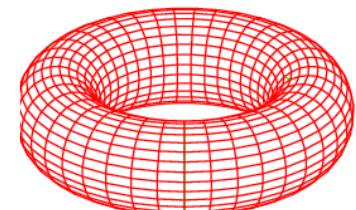
R^3



S^3



$R \times S^2$



$S^1 \times S^2$

Numerical precision and energy conservation

- GridRipper AMR: 4th order in space, RK4 in time.

- To check numerical precision, a conserved energy current is defined using the Kodama vector.

$$K^\alpha = \left(\frac{r_\rho}{\alpha\beta}, -\frac{r_\tau}{\alpha\beta}, 0, 0 \right),$$

$$J^\alpha = T^\alpha{}_\mu K^\mu,$$

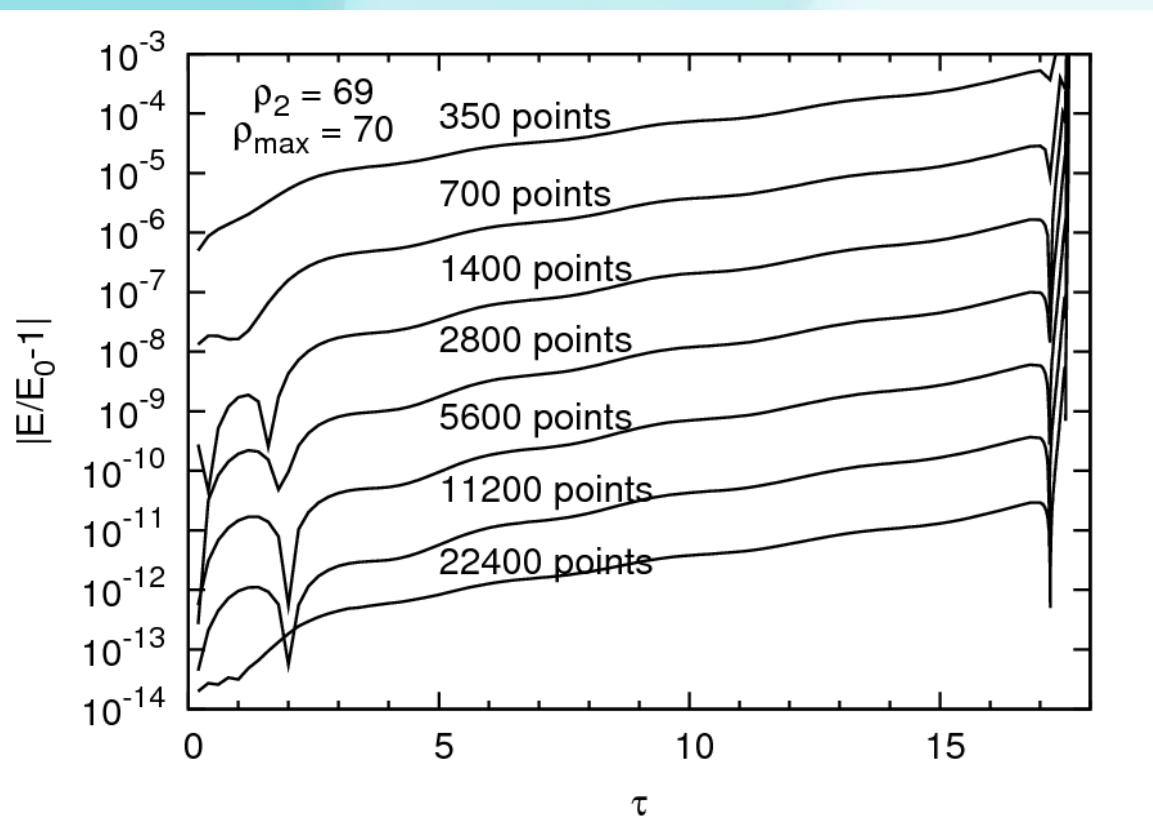
$$\partial_\mu J^\mu = 0 \Rightarrow \int_{\partial\Omega} J^\mu d^3\Sigma_\mu = 0.$$

- Results for a gravitationally collapsing massive scalar, with unit lapse

$$V = \frac{1}{2}\mu^2\psi^2$$

$$\mu = 0.8$$

$$\beta = 1$$

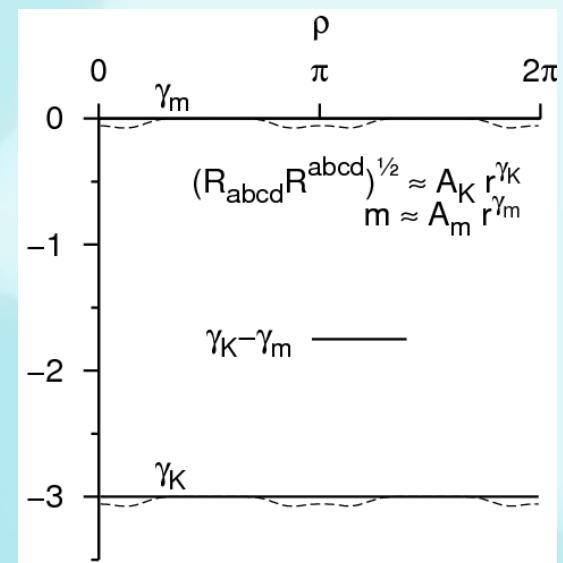
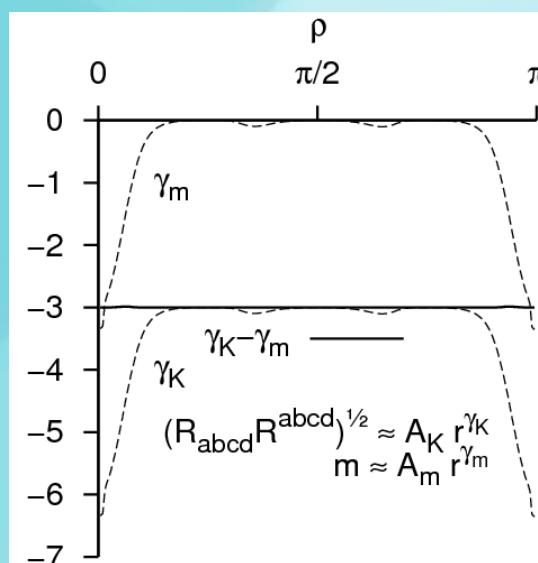
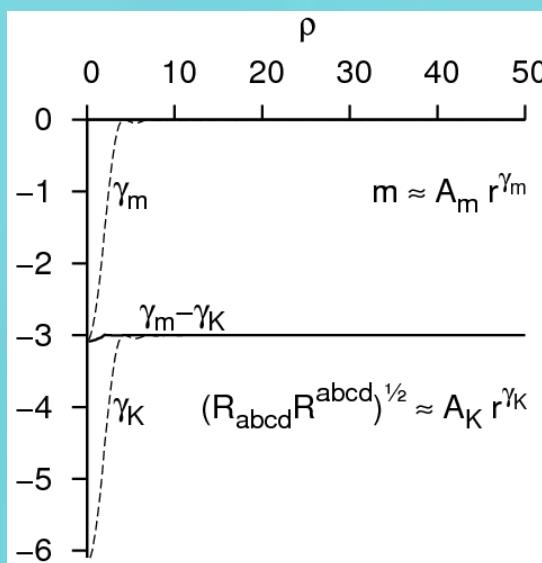
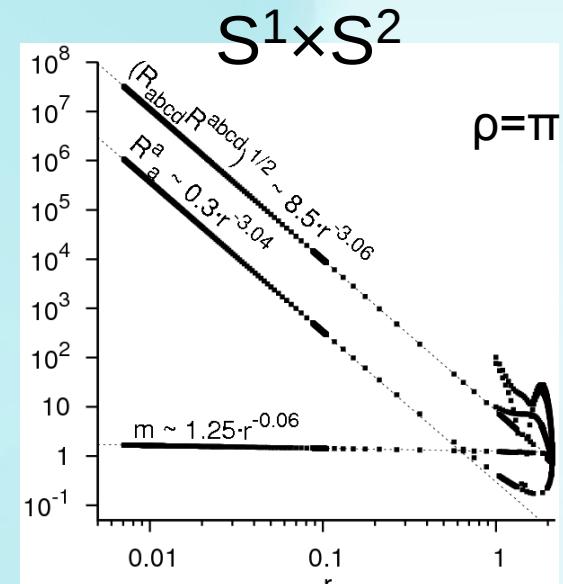
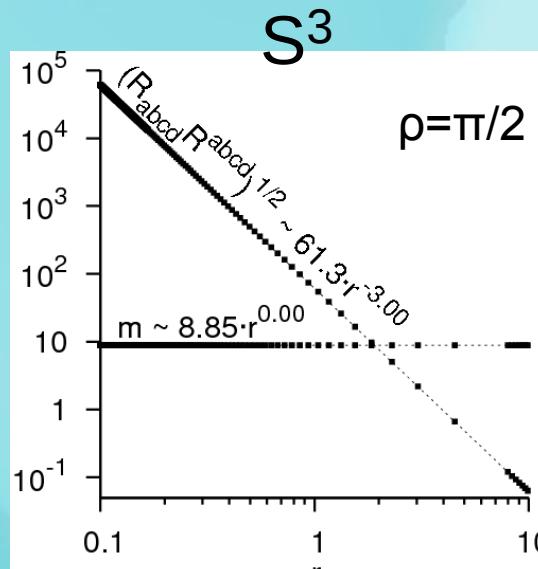
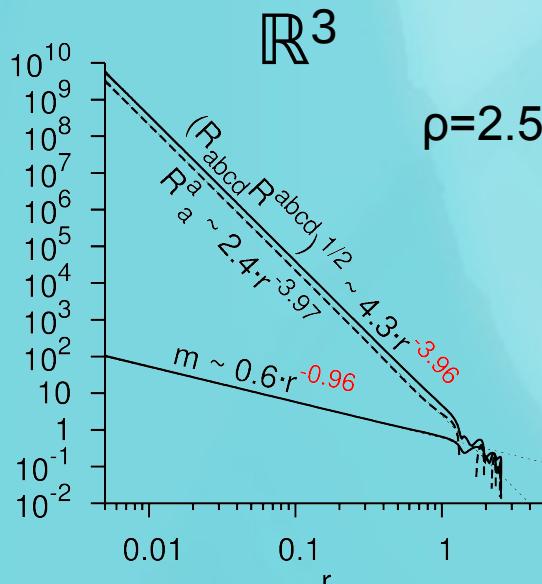


Critical exponents

Analytic result of Christodoulou (1991) for a massless scalar field in \mathbb{R}^3

$$(R_{abcd} R^{abcd})^{1/2} \geq 4\sqrt{2} m/r^3$$

S3-3d-sqrtK.sh



RMKI-Virgo csoport

Péter a csoport egyik meghatározó tagja

- Kifejezetten kerülte az olyan mechanikus feladatokat, amelyekben “csak” bonyolult algoritmusok alkalmazása történik, ugyanakkor a kreativitásnak semmi szerepe.
 - Script-eket készített, amelyek használatával a teljes GW adatanalízis futtatható volt (2009 jan.-febr.).
- Kidolgozta a kompakt kettősök poszt-Newtoni leírásban a mozgásprobléma megoldására, valamint a keltett gravitációs sugárzás meghatározására alkalmas numerikus kódot.

Kidder4test

Ami a fizikán túl van



Erice